Lesson 56

M70 B 14.1

Review of Sequences & Series.

Review: Sequences and Series

Arithmetic sequence:

- Add (or subtract = add negative) the same number *d* each time
- $a_n = a_1 + d(n-1)$

Geometric sequence

- Multiply (or divide = multiply by a fraction) the same number *r* each time
- $a_n = a_1 \cdot r^{n-1}$

Practice

Write the general term for each sequence.

1) $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

2) $2, \frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}$

3) $6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}$

4) $2, -1, -4, -7, -10$

Write in summation notation.

5) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

6) $2 + \frac{8}{3} + \frac{10}{3} + 4 + \frac{14}{3} + \frac{16}{3}$

7) $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8}$

8) $2 - 1 - 4 - 7 - 10 - 13 - 16 - 19 - 21$

9) Find a_{100} when $a_n = \frac{(-1)^{n+1} \cdot n^2}{4}$

10) Evaluate

$$\sum_{n=1}^{6} 3 \cdot 2^n$$

11) Evaluate S_3 when $a_n = 7n - 1$

12) Write the general term: $3, -2, -7, -12, -17$

13) Write in summation notation: $10 + 2 + \frac{2}{5} + \frac{2}{25}$

14) Find the first six terms of $a_n = 7\left(-\frac{2}{5}\right)^{n+1}$

Review: Sequences and Series

Write the general term:

(1) $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

$\times \frac{1}{3}$ must each time $\Rightarrow$ geometric

$a_1 = 2$

$r = \frac{1}{3}$

$a_n = a \cdot r^{n-1}$

$\boxed{a_n = 2 \cdot \left(\frac{1}{3}\right)^{n-1}}$ or $\boxed{a_n = \frac{2}{3^{n-1}}}$

(2) $2, \frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}$

rewrite with common denominator

$\frac{6}{3}, \frac{8}{3}, \frac{10}{3}, \frac{12}{3}, \frac{14}{3}$

$+\frac{2}{3}$ add each time $\Rightarrow$ arithmetic

$a_1 = 2$

$d = 2/3$

$a_n = a_1 + d(n-1)$

$a_n = 2 + \frac{2}{3}(n-1)$

$= 2 + \frac{2}{3}n - \frac{2}{3}$

$\boxed{a_n = \frac{2}{3}n + \frac{4}{3}}$

(3) $6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}$

$\times\left(-\frac{1}{2}\right)$ must each time $\Rightarrow$ geometric

$a_1 = 6$

$r = -1/2$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 6\left(-\frac{1}{2}\right)^{n-1}}$

or $\boxed{a_n = \frac{6}{(-2)^{n-1}}}$

(4) $2, -1, -4, -7, -10$

-3 Subtract each time $\Rightarrow$ arithmetic

$a_1 = 2$

$d = -3$

$a_n = a_1 + d(n-1)$

$= 2 + (-3)(n-1)$

$= 2 - 3n + 3$

$\boxed{a_n = -3n + 5}$

Write in summation notation

(5) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

4 terms

$\sum_{i=1}^{4} a_i$ $\quad a_i$ = general term

Note: This question matches (1)

$= \boxed{\sum_{i=1}^{4} 2\left(\frac{1}{3}\right)^{i-1}}$

or $\boxed{\sum_{n=1}^{4} 2\left(\frac{1}{3}\right)^{n-1}}$

(6) $= 2 + \frac{8}{3} + \frac{10}{3} + 4 + \frac{14}{3} + \frac{16}{3}$

6 terms

$\boxed{\sum_{n=1}^{6} \left(\frac{2}{3}n + \frac{4}{3}\right)}$

Note: This question matches (2)

(7) $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8}$

5 terms

$\boxed{\sum_{n=1}^{5} 6\left(\frac{-1}{2}\right)^{n-1}}$

Note: This question matches (3)

(8) $2 - 1 - 4 - 7 - 10 - 13 - 16 - 19 - 21$

9 terms

$$\boxed{\sum_{n=1}^{9} (-3n+5)}$$

Note: This question matches (4).

(9) Find a_{100} when $a_n = \dfrac{(-1)^{n+1} \cdot n^2}{4}$

subst $n=100$ $\quad a_{100} = \dfrac{(-1)^{100+1} \cdot (100)^2}{4}$

$$= \frac{(-1)^{101} \cdot 10000}{4}$$

odd power of (-1) is -1

$$= -1 \cdot 2500$$

$$= \boxed{-2500}$$

(10) Evaluate $\sum_{n=1}^{6} 3 \cdot 2^n$

$$= \underset{n=1}{3 \cdot 2^1} + \underset{n=2}{3 \cdot 2^2} + \underset{n=3}{3 \cdot 2^3} + \underset{n=4}{3 \cdot 2^4} + \underset{n=5}{3 \cdot 2^5} + \underset{n=6}{3 \cdot 2^6}$$

$$= 6 + 12 + 24 + 48 + 96 + 192$$

$$= \boxed{368}$$

(11) Evaluate S_3 when $a_n = 7n - 1$

$$S_3 = \sum_{n=1}^{3} (7n-1)$$

$$= \underset{n=1}{7(1)-1} + \underset{n=2}{7(2)-1} + \underset{n=3}{7(3)-1}$$

$$= 6 + 13 + 20$$

$$= \boxed{39}$$

(12) Write the general term $3, -2, -7, -12, -17$

-5 subtract 5 each time

arithmetic

$a_1 = 3$

$d = -5$

$a_n = a_1 + d(n-1)$

$= 3 - 5(n-1)$

$= 3 - 5n + 5$

$= \boxed{-5n + 8}$

(13) Write in summation notation. $10 + 2 + \frac{2}{5} + \frac{2}{25}$

$\times \frac{1}{5}$

- need general term

  mult by $\frac{1}{5}$ each time $\Rightarrow$ geometric

  $a_1 = 10$

  $r = \frac{1}{5}$

  $a_n = a_1 \cdot r^{n-1}$

  $a_n = 10\left(\frac{1}{5}\right)^{n-1}$

- 4 terms added

$$\boxed{\sum_{n=1}^{4} 10\left(\frac{1}{5}\right)^{n-1}}$$

(14) Find the first six terms of $a_n = 7\left(\frac{-2}{5}\right)^{n+1}$

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$7\left(\frac{-2}{5}\right)^{1+1}$	$7\left(\frac{-2}{5}\right)^{2+1}$	$7\left(\frac{-2}{5}\right)^{3+1}$	$7\left(\frac{-2}{5}\right)^{4+1}$	$7\left(\frac{-2}{5}\right)^{5+1}$	$7\left(\frac{-2}{5}\right)^{6+1}$
$7\left(\frac{-2}{5}\right)^{2}$	$7\left(\frac{-2}{5}\right)^{3}$	$7\left(\frac{-2}{5}\right)^{4}$	$7\left(\frac{-2}{5}\right)^{5}$	$7\left(\frac{-2}{5}\right)^{6}$	$7\left(\frac{-2}{5}\right)^{7}$
$7\left(\frac{4}{25}\right)$	$7\left(\frac{-8}{125}\right)$	$7\left(\frac{16}{625}\right)$	$7\left(\frac{-32}{3125}\right)$	$7\left(\frac{64}{15625}\right)$	$7\left(\frac{-128}{78125}\right)$

$$\boxed{\frac{28}{25}, \frac{-56}{125}, \frac{112}{625}, \frac{-224}{3125}, \frac{448}{15625}, \frac{-896}{78125}}$$